A Comparative Study of Constrained Multi-objective Evolutionary Algorithms on Constrained Multi-objective Optimization Problems

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***Abstract*—Solving constrained multi-objective optimization problems is a difficult task, it needs to simultaneously optimize multiple conflicting objectives and a number of constraints. This paper first reviews a number of popular constrained multi-objective evolutionary algorithms (CMOEAs) and twenty-three widely used constrained multi-objective optimization problems (CMOPs) (including CF1-10, CTP1-8, BNH, CONSTR, OSY, SRN and TNK problems). Then eight popular CMOEAs with simulated binary crossover (SBX) and differential evolution (DE) operators are selected to test their performance on the twenty-three CMOPs. The eight CMOEAs can be classified into domination-based CMOEAs (including ATM, IDEA, NSGA-II-CDP and SP) and decomposition-based CMOEAs (including CMOEA/D, MOEA/D-CDP, MOEA/D-SR and MOEA/D-IEpsilon). The comprehensive experimental results indicate that IDEA has the best performance in the domination-based CMOEAs and MOEA/D-IEpsilon has the best performance in the decomposition-based CMOEAs. Among the eight CMOEAs, MOEA/D-IEpsilon with both SBX and DE operators has the best performance on the twenty-three test problems.**

I. Introduction

The real-world optimization problems usually involve the simultaneous optimization of multiple conflicting objectives with a number of constraints. Without loss of generality, a constrained multi-objective optimization problem (CMOP) can be defined as follows [1]:

$$\begin{aligned} \text{minimize} \quad & \mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), \ldots, f_m(\mathbf{x}))^T \\ \text{subject to} \quad & g_i(\mathbf{x}) \geq 0, i = 1, \ldots, q \\ & h_j(\mathbf{x}) = 0, j = 1, \ldots, p \\ & \mathbf{x} \in \mathbb{R}^n \end{aligned} \tag{1}$$

where $\mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x}))^T \in \mathbb{R}^m$ is a m-dimensional objective vector, $g_i(\mathbf{x}) \geq 0$ defines i-th of q inequality constraints, $h_j(\mathbf{x}) = 0$ defines j-th of q equality constraints.

A solution $\mathbf{x}$ is feasible if it simultaneously meets $g_i(\mathbf{x}) \geq 0$ for each $i \in \{1, \ldots, q\}$ and $h_j(\mathbf{x}) = 0$ for each $j \in \{1, \ldots, p\}$. For two feasible solutions $\mathbf{x}^1$ and $\mathbf{x}^2$, solution $\mathbf{x}^1$ is said to dominate $\mathbf{x}^2$ if $f_i(\mathbf{x}^1) \leq f_i(\mathbf{x}^2)$ for each $i \in \{1, \ldots, m\}$ and $F(\mathbf{x}^1) \neq F(\mathbf{x}^2)$, denoted as $\mathbf{x}^1 \preceq \mathbf{x}^2$. For a feasible solution $\mathbf{x}^* \in \mathbb{R}^n$, if there is no other feasible solutions dominating $\mathbf{x}^*$, $\mathbf{x}^*$ is a Pareto optimal solution. The set of all the Pareto optimal solutions is called a Pareto Set (PS). The image of PS in the objective space is called a Pareto Front (PF), where $PF = \{\mathbf{F}(\mathbf{x}) \in \mathbb{R}^m | \mathbf{x} \in PS\}$.

When solving a CMOP, the objectives and constraints need to be optimized simultaneously. CMOEAs are very suitable for solving CMOPs, as they are able to obtain a set of non-dominated solutions in a single run. Apparently, the constraint handling mechanisms are critical for CMOEAs, which can be roughly classified into penalty functions, separation of constraints and objectives, multi-objective evolutionary algorithms (MOEAs) and hybrid methods.

In penalty functions, there are several different types, which include death penalty [2], static penalty [3], dynamic penalty [4], adaptive penalty [5–8] and so on. In the death penalty approach [2], when a certain solution violates any constraints, it is rejected and generated again. In the static penalty method [3], the penalty factors, which balance the searching between the feasible and infeasible regions, remain constant during the entire search process. In the dynamic penalty method [4], the penalty factors usually increase over time. The adaptive penalty method [5–8] takes feedbacks from the search process, such as the portion of feasible solutions and the average value of the constraint violations, and these feedbacks are utilized to adjust the penalty factors adaptively.

The main issue of the penalty functions of constraint handling is that the ideal penalty factors can not be known in advance for an arbitrary CMOP, thus tuning the penalty factors

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is a very tedious task.

In the separation of constraints and objectives, the objectives and constraints are handled separately. Representative approaches of this type include constraint dominance principle (CDP) [9], stochastic ranking (SR) [10] and epsilon constraint handling [11]. In CDP [9], three rules are applied to compare any two solutions. For solutions $\mathbf{x}^1, \mathbf{x}^2$, if $\mathbf{x}^1$ is feasible and $\mathbf{x}^2$ is infeasible, $\mathbf{x}^1$ is better than $\mathbf{x}^2$. If $\mathbf{x}^1$ and $\mathbf{x}^2$ are both infeasible, the one with the smaller constraint violation is better. If $\mathbf{x}^1$ and $\mathbf{x}^2$ are both feasible, the one dominating the other is better. In the SR method [10], the comparison of any two solutions is based on their objectives or constraints with a predefined probability. The epsilon constraint handling method [11] is similar to CDP, the only difference is that a solution is treated as a feasible solution if its constraint violation is less than a given threshold ϵ. when ϵ equals to zero, the epsilon constraint handling method is equivalent to CDP.

The constraint handling method of MOEAs transforms a CMOP to an unconstrained MOP by converting the constraints into an extra objective. Representative method is Cai and Wang's Method (CW) [12]. The in-feasibility driven evolutionary algorithms (IDEA) [13] also adopts this method to handle constraints. In its infeasible sub-population, the constraints are converted into an extra objective, and non-dominated ranking are utilized to sort the infeasible sub-population with $m+1$ objectives.

The hybrid methods of constraint handling usually adopt several methods together to handle constraints. The adaptive trade-off model (ATM) [14] adopts two mechanisms - MOEAs and adaptive penalty functions to handle constraints.

The remainder of this paper is organized as follows. Section II introduces a number of representative CMOEAs. Section III gives a brief review of the existing CMOPs. Section IV designs a comprehensive experiments to compare the existing CMOEAs, and Section VII concludes the paper.

II. RELATED WORKS OF CMOEAS

A CMOEA consists of two parts, one part is the constraint handling and the other part is the MOEA. In terms of MOEAs, the existing methods can be categorized into domination-based, decomposition-based and indicator-based approaches according to their selection approaches. However, there has been rare work conducted for indicator-based CMOEAs.

In the domination-based CMOEAs, non-dominated ranking is adopted to rank the solutions. Representative methods consist of ATM[14], IDEA[13], NSGA-II with constrained dominance principle (NSGA-II-CDP)[9], self-adaptive penalty (SP)[8].

In ATM [14], the working population is classified into three situations according to the feasibility proportion of the current population - the infeasible, semi-infeasible and feasible phases. In the infeasible situation, the constraints are converted to an extra objective in ATM. Therefore, a CMOP with m objectives and $p+q$ constraints is transformed into an unconstrained MOP with $m+1$ objectives. In the semi-infeasible situation, a penalty function is adopted to integrate the constraints in the m objectives, and non-dominated sorting is applied on the m integrated objectives. In the feasible situation, non-dominated sorting is adopted directly on the m objectives.

IDEA [13] explicitly maintains a small proportion of infeasible solutions close to the constraint boundaries during its evolution process. More specifically, it divides the population into feasible and infeasible sub-population. In the infeasible sub-population, the constraints are converted to an extra objective, and non-dominated ranking is adopted to rank the solutions with $m+1$ objectives. Then, a small proportion of infeasible solutions are selected firstly from the infeasible sub-population. The rest of solutions are successively selected from the feasible sub-population according to their non-dominated ranks.

NSGA-II-CDP [9] adopts CDP to handle constraints. In NSGA-II-CDP, feasible solutions have better non-dominated ranks than any infeasible solutions, and for two infeasible solutions, the one with the smaller overall constraint violation has a better rank. In fact, NSGA-II-CDP divides the population into one feasible and one infeasible sub-population. In the feasible sub-population, non-dominated ranking is executed on objectives directly to select solutions. In the infeasible sub-population, solutions are ranked based on their overall constraint violations. NSGA-II-CDP first selects offspring from the feasible sub-population, and then selects solutions from the infeasible sub-population until the number of offspring reaching the population's size.

SP [8] adopts an adaptive penalty function and a distance function to handle the constraints. These two functions vary dependent upon the objectives and the overall constraints violation of a solution. The fitness function of $i-th$ objective is the sum of its penalty and distance function. The proportion of feasible solutions in the population is adopted to balance the search preference between the feasible and infeasible regions.

In the decomposition-based CMOEAs, A CMOP is decomposed into a set of constrained single objective optimization subproblems, and these subproblems are solved in a collaborative way. Representative methods include CMOEA/D [15], MOEA/D-CDP [16], MOEA/D-SR [16] and MOEA/D-IEpsilon [17].

CMOEAD [15] embeds epsilon constraint handling approach in MOEA/D, and the epsilon value is set adaptively for comparison. For two solutions, if their constraint violations are both less than the epsilon value or they have the same constraint violations, the one with the better aggregation value is selected. Otherwise, the one with the smaller constraint violation is selected.

MOEA/D-CDP [16] adopts the CDP to handle constraints in the framework of MOEA/D. For two feasible solutions, the one with the better aggregation value is selected. For two infeasible solutions, the one with the smaller constraint violation is selected. For a feasible and an infeasible solution, the feasible one is selected.

MOEA/D-SR [16] applies SR to handle constraints in the framework of MOEA/D. In MOEA/D-SR, a parameter p_f decides the probability of using the objectives to compare two solutions. For two solutions, if a random is less than p_f, the one with better aggravation value is selected into next generation. If the random is greater than p_f, the solutions selection is similar to the MOEA/D-CDP. If $p_f = 0$, MOEA/D-SR is equivalent to MOEA/D-CDP.

MOEA/D-IEpsilon [17] applies an improved epsilon constraint handling method in the framework of MOEA/D. Unlike the original epsilon setting method with a decreasing epsilon level, the epsilon level in MOEA/D-IEpsilon is increased if the proportion of feasible solutions in the current population is greater than a predefined threshold.

III. The existing CMOPs

Currently, the common widely tested CMOPs include CF [18], CTP[1], BNH [19], CONSTR [1], OSY [20], SRN [21] and TNK [22] problems. The feasible regions of these problems except OSY are shown in the green part of Fig. 1. The PFs without considering constraints of these problems except OSY are shown in the red part of Fig. 1. For OSY problem, it is too difficult to sample the feasible regions in the objective space. Only the PF of OSY problem is plotted as shown in Fig. 1(u).

For CF1-3 and CF8-10, their PFs are a part of their unconstrained PFs as shown in Fig. 1(a)-(c) and Fig. 1(h)-(j). The constraints of CF1-3 and CF8-10 have the difficulties in the entire search space. However, the PFs of these problems can be achieved by MOEAs without considering any constraints. After MOEAs get the unconstrained PFs of these problems, the set of feasible solutions are their constrained PFs. For CF4-7, their PFs are partly constructed by their unconstrained PFs and partly constructed by their constraint boundaries as shown in Fig. 1(d)-(g). The constraints of these problems have the difficulties near their PFs.

For CTP test instance, the constraints are constructed by each objective. The PF of CTP1 consists of two parts as shown in Fig. 1(k), one part is on the boundaries of constraints, and the other one is a part of the unconstrained PF of CTP1. The constraints of CTP1 have the difficulties near its PF. For CTP2-CTP6 and CTP8, their PFs locate at the boundaries of their constraints as shown in Fig. 1(l)-(p) and Fig. 1(r). For CTP7, the PF is a part of its unconstrained PF as shown in Fig. 1(q). The constraints of CTP2-8 have the difficulties in the entire search space. For BNH, CONSTR and SRN, the PFs are partly constructed by their unconstrained PFs and partly constructed by their boundaries of constraints as shown in Fig. 1(s), Fig. 1(t) and Fig.1(v). The Constraints of these problems make their original unconstrained PFs partially feasible. The PFs of OSY and TNK all locate on their constraint boundaries. As the objectives for TNK are not conflicting with each other, its PF without constraints is degenerated into only one point, as shown in Fig.1(w).

Based on the above analysis, the existing CMOPs can be helpful to evaluate the performance of algorithms on CMOPs with different properties. However, some limitations may also exist to hinder their further performance on testing algorithms, as follows. For CTP test problems, the first objective is so simple that the diversity of population can be easily maintained, which may not help to evaluate the ability of maintaining diversity for a CMOEA. For CF test instance, the objective functions are so difficult to evaluate the effect of constraint handling mechanisms. For BNH, CONSTR, OSY, SRN and TNK, the dimension of decision vector is too low to evaluate the performance of CMOEAs. In the above CMOPs, the proportion of feasible regions are relatively large, and the constraint handling mechanism may not play any important roles on the run time of a algorithm.

IV. Experimental results

A. Experimental Setting

To verify the advantages and disadvantages of the existing CMOEAs, eight CMOEAs with simulated binary crossover (SBX) and differential evolution (DE) crossover are tested on the twenty-three CMOPs. The detailed parameters of these algorithms are listed as follows:

1) Setting for reproduction operators: The mutation probability $Pm = 1/n$ (n is the number of decision variables). For the polynomial mutation operator, the distribution index is set to 20. For the SBX operator, the distribution index is set to 20. For the DE operator, $CR = 0.9$ and $f = 0.5$.
2) Population size: for two-objective CMOPs, $N = 200$, for three-objective CMOPs, $N = 300$.
3) Number of runs and stopping condition: Each algorithm runs for 30 times independently. For two-objective CMOPs, the algorithm stops until 100000 function evaluations. For three-objective CMOPs, the algorithm stops until 150000 function evaluations.
4) Neighborhood size: $T = 0.1N$.
5) Probability of selecting subproblems in the neighborhood: $\delta = 0.9$.
6) The maximal number of solution replacements: $nr = 2$.
7) Other parameters for MOEA/D-IEpsilon: $\alpha = 0.8$, $T_c = 400$, $cp = 2$, $\tau = 0.1$ and $\theta = 0.2NI$. NI is the number of infeasible solutions in the initial population.

B. Performance Metric

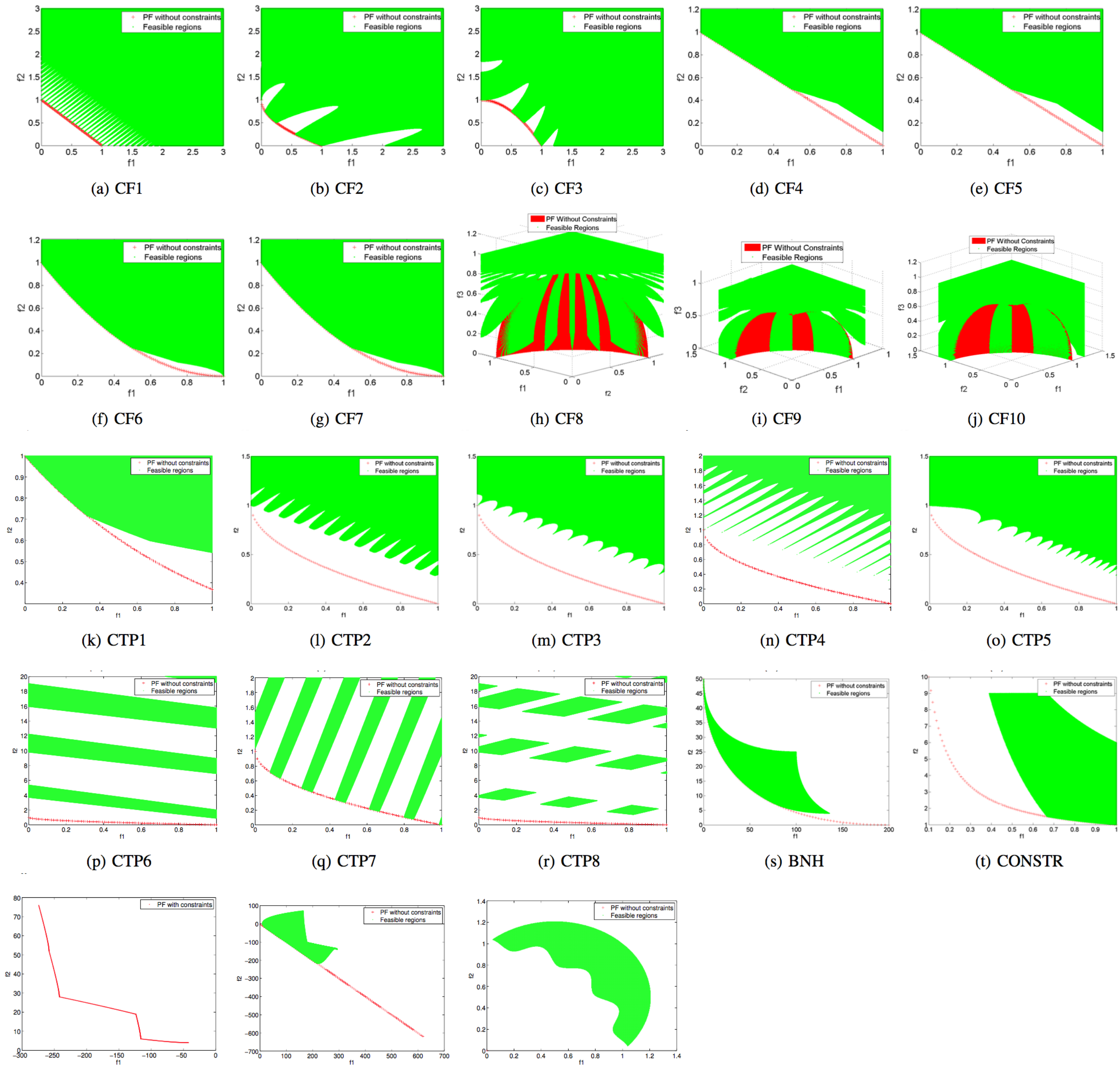
To measure the performance of ATM, CMOEA/D, IDEA, MOEA/D-CDP, MOEA/D-SR, NSGA-II-CDP, SP and MOEA/D-IEpsilon, the inverted generation distance (IGD)[23] metric is adopted. The definitions of IGD is listed as follows:

- **Inverted Generational Distance** (IGD):

The IGD metric reflects the performance of convergence and diversity simultaneously. The detailed definition is listed as follows:

$$\begin{cases} IGD(P^*, A) = \frac{\sum\limits_{y^* \in P^*} d(y^*, A)}{|P^*|} \\ \\ d(y^*, A) = \min\limits_{y \in A}\{\sqrt{\sum_{i=1}^{m}(y_i^* - y_i)^2}\} \end{cases} \quad (2)$$

where P^* is the ideal Pareto front set, A is an approximate Pareto front set achieved by an algorithm. The values of IGD



(a) CF1 (b) CF2 (c) CF3 (d) CF4 (e) CF5
(f) CF6 (g) CF7 (h) CF8 (i) CF9 (j) CF10
(k) CTP1 (l) CTP2 (m) CTP3 (n) CTP4 (o) CTP5
(p) CTP6 (q) CTP7 (r) CTP8 (s) BNH (t) CONSTR
(u) OSY (v) SRN (w) TNK

Fig. 1. Illustrations on the feasible regions of CF1-10, CTP1-8, BNH, CONSTR, OSY, SRN and TNK problems.

denotes the distance between P^* and A. For CMOPs with two objectives, 1000 points are sampled in the PFs uniformly to construct the set of P^*. For CMOPs with three objectives, 10000 points are sampled in the PFs uniformly. It is worth noting that the smaller value of IGD represents the better performance of both diversity and convergence.

C. Experimental Discussion

Table I shows the mean and standard deviation values of IGD of the eight tested CMOEAs with the SBX operator on CF1-10, CTP1-8, BNH, CONSTR, OSY, SRN and TNK problems. For CF1 and CF7-8, MOEA/D-IEpsilon is better than the other seven CMOEAs. IDEA performs better than the other seven CMOEAs on CF2-CF6. CMOEA/D works better than the other seven CMOEAs on CF9. For CF10, all the tested CMOEAs except MOEA/D-IEpsilon have infinite mean values of IGD, which means that these CMOEAs sometimes can not achieve any feasible solutions during the 30 independent runs. However, MOEA/D-IEpsilon can always get feasible solutions in the 30 independent runs. For CTP1, CTP5 and OSY problems, ATM gets the best results among the eight tested CMOEAs. MOEA/D-IEpsilon performs better than the other seven CMOEAs on CTP2, CTP4, CTP6-8, CONSTR, SRN and TNK problems. For CTP3, SP has the best performance among the eight CMOEAs. For BNH problem, NSGA-II-CDP is better than the other seven CMOEAs.

To rank the eight CMOEAs with SBX operator, we set the total points for each CMOEA. If a CMOEA is better than other seven CMOEAs on a certain problem, then it gets one point. On the twenty-three test instances, MOEA/D-CDP and MOEA/D-SR get zero point. ATM, CMOEA/D, NSGA-II-CDP and SP get one point. IDEA gets seven points. MOEA/D-IEpsilon gets twelve points. Therefore, MOEA/D-IEpsilon has the best performance among the eight tested CMOEAs with the SBX operator.

Table II shows the mean and standard deviation values in terms of IGD obtained by the eight tested CMOEAs with the DE crossover operator on the twenty-three problems. For CF1, MOEA/D-CDP performs better than the other seven CMOEAs, and MOEA/D-SR is better than the other seven CMOEAs on CF2-3. For CF4-5 and CF7-10, MOEA/D-IEpsion has better performance than the other seven CMOEAs. For CF6, SP works better than the other seven CMOEAs. For CTP1 and BNH problem, NSGA-II-CDP performs better than the other seven CMOEAs. For CTP2-4, CTP6-8, SRN and TNK problems, MOEA/D-IEpsilon has the best performance among the eight CMOEAs. CMOEA/D is better than the other seven CMOEAs on CTP5. IDEA works better than other CMOEAs on CONSTR and OSY problems. On the twenty-three test instances, ATM get zero point. CMOEA/D, MOEA/D-CDP and SP get one point. IDEA, MOEA/D-SR, and NSGA-II-CDP get two points. MOEA/D-IEpsilon gets fourteen points. Therefore, MOEA/D-IEpsilon has the best performance among the eight tested CMOEAs with the DE operator.

From Table I and Table II, it can be observed that the best mean IGD values obtained by the eight CMOEAs with DE operator are better than their SBX versions on CF1-9 test instances. However, for CTP2-8 test instances, the best mean IGD values obtained by CMOEAs with SBX operator are better than their DE versions. We can summary that DE operator works better than SBX operator on CF test problems, and SBX operator works better than DE operator on CTP test problems.

For OSY, IDEA with DE and SBX operators is always better than the other seven CMOEAs. One possible reason is that the PF of OSY locates on the boundaries of the constraints, and IDEA introduces a small portion of infeasible solutions to its population, which has the ability to help to search the solutions on the boundaries of the constraints.

Among the eight tested CMOEAs, IDEA has the best performance among the domination-based CMOEAs, and MOEA/D-IEpsilon has the best performance among the decomposition-based CMOEAs. Both CMOEAs explicitly increase the search preference in the infeasible regions. We can summary that increasing the searching preference appropriately in the infeasible regions can help to enhance the performance of a CMOEA.

From the above experiment observation, we can conclude that MOEA/D-IEpsilon has the best performance among the eight tested CMOEAs with both DE and SBX operators on the twenty-three test problems.

V. Conclusion

In this paper, we first review the existing CMOPs, and observe that the proportion of feasible regions of these problems are relatively large, which may not be suitable to test the effect of the constraint handling approaches in CMOEAs. Then eight popular CMOEAs with SBX and DE operators are tested on the twenty-three problems. The experimental results indicate that IDEA has the best performance in the domination-based CMOEAs, and MOEA/D-IEpsilon has the best performance among the eight tested CMOEAs. A common characteristic of IDEA and MOEA/D-IEpsilon is that both of them keep a number of infeasible solutions during their evolutionary process, which can be concluded that maintaining a small portion of infeasible solutions in the working population appropriately can help to enhance the performance of a CMOEA when solving CMOPs. Furthermore, the experimental results show that the DE operator has better performance than the SBX operator on the CF benchmarks, and SBX operator works better than DE operator on the CTP benchmarks.

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TABLE I
PERFORMANCE OF ATM, CMOEA/D, IDEA, MOEA/D-CDP, MOEA/D-SR, NSGA-II-CDP, SP AND MOEA/D-IEPSILON WITH SBX OPERATOR ON CF, CTP, BNH, CONSTR, OSY, SRN AND TNK PROBLEMS IN TERMS OF THE MEAN AND STANDARD DEVIATION VALUES OF IGD

Problem		ATM	CMOEA/D	IDEA	MOEA/D-CDP	MOEA/D-SR	NSGA-II-CDP	SP	MOEA/D-IEpsilon
CF1	mean	5.786E-02	1.549E-02	2.035E-02	1.440E-02	1.954E-02	5.964E-02	5.836E-02	**6.057E-03**
	std	8.845E-03	4.629E-03	2.662E-03	3.676E-03	4.250E-03	8.278E-03	1.078E-02	1.358E-03
CF2	mean	1.472E-01	1.254E-01	**5.229E-02**	1.184E-01	1.235E-01	1.605E-01	1.510E-01	1.043E-01
	std	3.560E-02	5.179E-02	3.819E-02	6.120E-02	6.788E-02	3.815E-02	4.388E-02	4.620E-02
CF3	mean	5.060E-01	3.610E-01	**2.043E-01**	3.255E-01	3.827E-01	5.414E-01	5.516E-01	3.148E-01
	std	1.465E-01	1.307E-01	6.093E-02	1.087E-01	1.425E-01	1.549E-01	1.269E-01	1.252E-01
CF4	mean	1.478E-01	1.772E-01	**8.301E-02**	1.930E-01	1.797E-01	1.558E-01	1.581E-01	1.176E-01
	std	3.145E-02	1.010E-01	2.332E-02	1.231E-01	8.271E-02	4.692E-02	4.719E-02	3.119E-02
CF5	mean	3.885E-01	3.653E-01	**2.645E-01**	3.892E-01	3.359E-01	3.779E-01	4.217E-01	2.911E-01
	std	1.320E-01	1.220E-01	1.325E-01	1.206E-01	1.201E-01	1.144E-01	1.028E-01	1.332E-01
CF6	mean	1.221E-01	1.242E-01	**6.602E-02**	1.217E-01	1.456E-01	1.188E-01	1.335E-01	1.336E-01
	std	4.025E-02	7.640E-02	3.116E-02	6.963E-02	6.211E-02	3.725E-02	4.526E-02	7.205E-02
CF7	mean	3.755E-01	4.681E-01	2.848E-01	3.862E-01	3.589E-01	4.150E-01	3.696E-01	**2.481E-01**
	std	9.599E-02	1.804E-01	1.246E-01	1.544E-01	1.323E-01	1.098E-01	6.571E-02	8.808E-02
CF8	mean	2.574E-01	Inf	2.000E-01	Inf	Inf	1.781E+00	2.825E-01	**9.945E-02**
	std	4.353E-02	NaN	3.339E-02	NaN	NaN	4.824E+00	2.315E-02	2.783E-02
CF9	mean	2.030E-01	**1.060E-01**	1.248E-01	1.169E-01	1.105E-01	1.900E-01	1.974E-01	1.070E-01
	std	1.366E-02	1.242E-02	1.031E-02	2.784E-02	9.937E-03	9.297E-03	2.193E-02	8.293E-03
CF10	mean	Inf	Inf	Inf	Inf	Inf	Inf	Inf	**2.974E-01**
	std	NaN	NaN	NaN	NaN	NaN	NaN	NaN	1.669E-01
CTP1	mean	**6.681E-03**	1.152E-01	5.043E-02	1.293E-01	8.657E-02	6.989E-03	1.276E-02	2.811E-01
	std	9.843E-03	7.531E-02	3.041E-02	7.988E-02	6.982E-02	1.299E-02	1.816E-02	6.573E-03
CTP2	mean	4.575E-03	9.847E-03	1.381E-03	1.421E-02	1.656E-02	4.757E-03	4.467E-03	**1.253E-03**
	std	1.719E-03	3.883E-02	7.894E-05	6.330E-02	6.313E-02	2.089E-03	1.625E-03	1.332E-04
CTP3	mean	1.350E-02	3.009E-02	1.430E-02	3.654E-02	4.508E-02	1.456E-02	**1.322E-02**	1.364E-02
	std	1.845E-03	7.485E-02	2.608E-03	6.333E-02	6.336E-02	2.240E-03	2.181E-03	1.772E-03
CTP4	mean	9.407E-02	1.198E-01	1.848E-01	1.860E-01	1.721E-01	1.112E-01	1.131E-01	**9.148E-02**
	std	1.381E-02	6.240E-02	1.451E-01	1.287E-01	1.294E-01	4.229E-02	5.013E-02	1.259E-02
CTP5	mean	9.629E-03	1.069E-02	**4.245E-03**	9.832E-03	3.079E-02	9.239E-03	9.633E-03	9.578E-02
	std	2.677E-03	3.214E-03	1.253E-03	3.186E-03	1.431E-02	2.625E-03	3.159E-03	5.979E-03
CTP6	mean	6.817E-03	8.636E-03	7.582E-03	8.647E-03	3.030E-02	7.040E-03	6.924E-03	**4.264E-03**
	std	5.807E-04	1.150E-03	2.372E-04	1.008E-03	5.368E-03	8.138E-04	3.937E-04	1.835E-04
CTP7	mean	6.492E-04	1.648E-03	8.319E-04	1.648E-03	1.593E-03	6.486E-04	6.503E-04	**4.350E-04**
	std	2.307E-05	3.306E-06	2.886E-05	6.735E-06	2.627E-05	1.604E-05	2.960E-05	1.353E-05
CTP8	mean	3.644E-03	9.490E-03	3.764E-03	1.586E-01	2.495E-02	4.181E-03	1.357E-02	**3.093E-03**
	std	8.085E-04	8.316E-04	6.309E-04	8.192E-01	4.315E-03	2.045E-03	5.453E-02	2.724E-04
BNH	mean	2.884E-01	1.329E+00	3.434E-01	1.330E+00	1.328E+00	**2.881E-01**	2.885E-01	3.911E-01
	std	1.469E-02	5.557E-03	1.628E-02	5.472E-03	5.769E-03	1.805E-02	1.516E-02	1.166E-01
CONSTR	mean	3.005E-02	2.327E-02	1.263E-02	2.322E-02	6.813E-02	3.293E-02	3.243E-02	**8.326E-03**
	std	1.151E-02	1.899E-04	3.140E-04	1.875E-04	1.341E-02	1.850E-02	1.551E-02	9.960E-04
OSY	mean	1.271E+01	1.411E+01	**1.523E+00**	1.380E+01	2.069E+01	1.457E+01	3.031E+00	3.864E+00
	std	2.741E+00	4.843E+00	9.758E-01	4.057E+00	7.031E+00	7.451E+00	1.364E+00	5.603E-01
SRN	mean	5.330E-01	1.022E+00	6.868E-01	1.017E+00	9.731E-01	5.318E-01	5.310E-01	**3.535E-01**
	std	1.667E-02	4.925E-02	1.980E-02	4.866E-02	4.767E-02	2.051E-02	1.437E-02	5.892E-03
TNK	mean	1.401E-02	2.564E-03	3.188E-03	2.548E-03	3.432E-02	2.124E-02	1.397E-02	**2.209E-03**
	std	6.581E-03	6.414E-05	8.024E-05	5.091E-05	8.159E-03	1.223E-02	6.101E-03	9.078E-05
Total Points		1	1	7	0	0	1	1	12

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TABLE II
PERFORMANCE OF ATM, CMOEA/D, IDEA, MOEA/D-CDP, MOEA/D-SR, NSGA-II-CDP, SP AND MOEA/D-IEPSILON WITH DE OPERATOR ON CF, CTP, BNH, CONSTR, OSY, SRN AND TNK PROBLEMS IN TERMS OF THE MEAN AND STANDARD DEVIATION VALUES OF IGD

Problem		ATM	CMOEA/D	IDEA	MOEA/D-CDP	MOEA/D-SR	NSGA-II-CDP	SP	MOEA/D-IEpsilon
CF1	mean	8.561E-03	3.480E-03	6.853E-03	**3.070E-03**	2.307E-02	8.687E-03	1.002E-02	5.127E-03
	std	2.532E-03	8.315E-04	1.779E-03	8.381E-04	4.798E-03	4.147E-03	6.302E-03	9.432E-04
CF2	mean	8.727E-02	1.303E-02	1.630E-02	9.885E-03	**3.300E-03**	9.230E-02	9.413E-02	1.414E-02
	std	2.378E-02	2.135E-02	3.634E-03	2.061E-02	1.405E-03	2.917E-02	2.338E-02	2.146E-02
CF3	mean	5.375E-01	2.032E-01	4.357E-01	2.151E-01	**1.931E-01**	5.378E-01	5.382E-01	2.331E-01
	std	1.222E-01	8.802E-02	5.552E-02	9.091E-02	9.376E-02	1.225E-01	1.358E-01	1.474E-01
CF4	mean	8.515E-02	2.739E-02	5.571E-02	3.387E-02	3.779E-02	8.044E-02	8.390E-02	**2.725E-02**
	std	2.923E-02	9.363E-03	5.376E-03	2.208E-02	2.844E-02	1.333E-02	2.231E-02	9.170E-03
CF5	mean	6.311E-01	3.003E-01	4.068E-01	2.903E-01	2.862E-01	6.405E-01	6.152E-01	**2.494E-01**
	std	1.332E-01	1.159E-01	1.066E-01	1.221E-01	1.257E-01	1.268E-01	1.227E-01	1.092E-01
CF6	mean	3.602E-02	6.388E-02	4.796E-02	5.761E-02	6.423E-02	4.295E-02	**3.202E-02**	6.459E-02
	std	9.862E-03	2.313E-02	2.575E-02	2.621E-02	3.227E-02	2.439E-02	6.212E-03	3.588E-02
CF7	mean	6.335E-01	2.701E-01	2.536E-01	2.903E-01	2.753E-01	6.043E-01	6.506E-01	**2.076E-01**
	std	1.946E-01	1.381E-01	8.513E-02	1.652E-01	1.726E-01	1.899E-01	1.793E-01	8.501E-02
CF8	mean	3.032E-01	1.177E-01	2.324E-01	Inf	Inf	4.798E-01	3.843E-01	**5.112E-02**
	std	7.533E-02	8.344E-03	4.197E-02	NaN	NaN	1.031E-01	4.047E-02	1.030E-02
CF9	mean	2.070E-01	7.974E-02	1.267E-01	7.794E-02	7.727E-02	1.939E-01	1.869E-01	**4.914E-02**
	std	5.243E-02	1.560E-02	1.136E-02	1.483E-02	1.654E-02	4.078E-02	3.566E-02	6.259E-03
CF10	mean	Inf	Inf	Inf	Inf	Inf	Inf	Inf	**3.007E-01**
	std	NaN	NaN	NaN	NaN	NaN	NaN	NaN	1.707E-01
CTP1	mean	2.169E-03	2.397E-03	2.516E-03	2.622E-03	5.040E-03	**2.132E-03**	2.158E-03	2.751E-01
	std	7.046E-05	6.276E-05	5.630E-05	1.066E-03	9.629E-04	6.155E-05	5.940E-05	8.501E-03
CTP2	mean	1.115E-02	3.092E-03	4.472E-03	2.597E-03	5.374E-03	1.040E-02	1.095E-02	**1.450E-03**
	std	3.401E-03	1.043E-03	8.611E-04	5.609E-04	1.380E-03	3.209E-03	2.692E-03	7.098E-04
CTP3	mean	3.282E-02	2.015E-02	3.143E-02	1.862E-02	3.407E-02	3.530E-02	3.522E-02	**1.563E-02**
	std	4.581E-03	4.235E-03	3.280E-03	2.984E-03	5.092E-03	4.618E-03	4.542E-03	2.755E-03
CTP4	mean	1.550E-01	1.168E-01	1.503E-01	1.169E-01	1.579E-01	1.513E-01	1.501E-01	**1.065E-01**
	std	2.038E-02	1.270E-02	2.169E-02	1.439E-02	1.964E-02	2.150E-02	1.280E-02	1.483E-02
CTP5	mean	2.220E-02	**4.548E-03**	9.175E-03	5.699E-03	2.364E-02	2.217E-02	2.097E-02	1.039E-01
	std	7.238E-03	1.305E-03	2.127E-03	3.592E-03	1.152E-02	6.639E-03	5.588E-03	1.171E-02
CTP6	mean	2.004E-02	7.843E-03	1.409E-02	8.025E-03	1.712E-02	2.011E-02	2.041E-02	**4.705E-03**
	std	4.022E-03	3.465E-04	1.098E-03	8.080E-04	2.873E-03	3.826E-03	5.557E-03	2.929E-04
CTP7	mean	6.338E-04	1.641E-03	7.933E-04	1.641E-03	1.433E-03	6.353E-04	6.297E-04	**4.752E-04**
	std	1.526E-05	5.306E-06	1.569E-05	5.333E-06	4.866E-05	1.812E-05	1.753E-05	1.811E-05
CTP8	mean	1.416E-02	8.577E-03	1.197E-02	8.494E-03	2.018E-02	2.451E-02	1.498E-02	**3.303E-03**
	std	5.982E-03	1.311E-03	4.686E-03	4.215E-04	8.057E-03	5.396E-02	6.527E-03	4.907E-04
BNH	mean	2.822E-01	1.332E+00	3.373E-01	1.330E+00	1.331E+00	**2.787E-01**	2.851E-01	4.594E-01
	std	1.536E-02	7.485E-03	2.029E-02	5.573E-03	7.140E-03	1.393E-02	1.568E-02	1.109E-01
CONSTR	mean	1.221E-02	2.328E-02	**1.088E-02**	2.326E-02	4.517E-02	1.359E-02	1.216E-02	1.281E-02
	std	3.687E-03	5.284E-05	2.720E-04	5.696E-05	7.173E-03	7.743E-03	6.314E-03	1.288E-03
OSY	mean	1.160E+01	1.159E+01	**2.745E+00**	1.195E+01	1.922E+01	1.136E+01	1.147E+01	4.306E+00
	std	1.038E+00	3.792E+00	1.726E+00	4.766E+00	4.126E+00	1.354E+00	8.604E-01	8.674E-01
SRN	mean	5.076E-01	1.050E+00	6.531E-01	1.080E+00	1.060E+00	5.131E-01	5.114E-01	**3.561E-01**
	std	1.154E-02	5.586E-02	2.367E-02	4.576E-02	5.712E-02	1.775E-02	1.309E-02	8.259E-03
TNK	mean	4.325E-03	2.522E-03	2.524E-03	2.526E-03	2.689E-02	8.087E-03	4.115E-03	**1.822E-03**
	std	1.008E-03	4.128E-05	7.433E-05	3.603E-05	3.246E-03	4.468E-03	9.478E-04	4.465E-05
Total Points		0	1	2	1	2	2	1	14

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